2. Problem sheet for Set Theory, Winter 2012

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Problem 3. (Ordered pairs)

    a) Show that $\langle x, y \rangle := \{\{x, \emptyset\}, \{y, \{\{\emptyset\}\}\}\}$ satisfies the fundamental property of ordered pairs.

    b) Does $\langle x, y \rangle := \{x, \{y, \emptyset\}\}$ satisfy the fundamental property of ordered pairs?

Problem 4. (Class functions) Suppose $F \colon A \to B$, $F' \colon A \to B$ are functions and X, Y are class terms. Show that

    a) $F^{-1}[X] \cap F^{-1}[Y] = F^{-1}[X \cap Y]$.

    b) $F = F'$ if and only if $\forall x \in A \ F(x) = F'(x)$.

    c) F is injective if and only if there is a class term G with $F \circ G = id_{range(F)}$.

Problem 5. (Equinumerous sets) Define a relation $\sim$ on V by

$$x \sim y \leftrightarrow \exists f \ f \colon x \leftrightarrow y.$$

One says that x and y are *equinumerous* or *equipollent*.

    a) Show that $\sim$ is an equivalence relation on V.

    b) What are the equivalence classes of $\emptyset$ and $\{\emptyset\}$?

Problem 6. Prove

    a) $\forall x \ (x \neq \emptyset \to \bigcap x \in V)$.

    b) $\forall x \ dom(x) \in V$.

    c) $\{\{x\}| \ x \in V\} \notin V$.

There are 6 points for each problem. Please hand in your solutions on Monday, October 22 before the lecture.